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CONNECTICUT UNIV STORRS DEPT OF CIVIL ENGINEERING
A PRELIMINARY STUDY OF HYDROELASTIC BEHAVIOR OF FAIRED TOWLINES--ETC(U)
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The University of Connecticut
Civil Engineering Department
Storrs, Connecticut

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Technical Report

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A Preliminary Study of Hydroelastic Behavior
of Faired Towlines

by

J. D. Lin

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January 29, 1965

Prepared Under

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New London, Connecticut

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The University of Connecticut
Civil Engineering Department

6 A Preliminary Study of Hydroelastic Behavior
of Paired Towlines.

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Abstract

→ A preliminary study of the hydroelastic behavior of a faired towline consists of a literature survey pertinent to the dynamics of a towline system, an examination of the stability of a sectional fairing in the light of the existing theories and experimental evidence of hydroelasticity, and a simplified calculation of the viscous flow past the AN/SQA-10 Fleet Type Towline with the USNUSL Sectional Fairing (the mid-section of the fairing is DTMB Hydrofoil Shape No. 7) in relation to the self-excited instability induced by the viscous wake. ←
The classical flutter is found to be unlikely for a section of the similar characteristics. The self-excited instability requires a more detailed and rigorous calculation of the flow field; however, the rough estimation indicates that the section can be operated at a speed of 30 knots normal to the fairing without a serious separation of the boundary layer from the fairing. The separation near the mid-body resulting a thick viscous wake is believed to cause the incipient of this instability. A further study and a refined calculation of viscous flow field is recommended for the prediction of separation as well as a initial step toward the calculation of the drag coefficient for faired towlines at any Reynolds number.

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Notations and Symbols

a	dimensionless distance between mid-chord and elastic axis, in half-chord length
b	half-chord length in feet
c	chord length in feet
G_p	pressure coefficient
d	thickness of body or diameter of circular cylinder in feet
f	frequency in cps
h	translational displacement in feet
H	shape factor ($= \delta^*/\theta$)
h_o, α_o	amplitude functions for translational and rotational oscillation respectively
i	imaginary number ($= \sqrt{-1}$)
I_α	mass polar moment of inertia per unit length about the elastic axis
k	reduced frequency ($= \omega b/U$)
K_h, K_α	spring constants in translation and rotation
L, M_y	force and moment per unit length at the elastic axis
m	mass per unit length in slugs/ft
p	pressure in psf or psi
q	the strength of source
R	Reynolds number ($= Ud/\nu$)
S	Strouhal number ($= \frac{fd}{U}$)
S_α	first moment of mass per unit length about the elastic axis
t	parameter or time in seconds
U	velocity of free stream in fps
u_s, v_s	velocity components due to symmetrical thickness in fps
u_c, v_c	velocity components due chamber in fps
U_F	flutter speed in fps

w	weight per unit length in lbs/ft
y_s	shape function of symmetrical thickness in feet
z	complex number ($= x + iy$)
α	rotational angle or angle of attack in radians
Γ	shape factor ($= \frac{\theta}{U} \frac{dU}{dx} (\frac{U\theta}{\gamma})^{\frac{1}{2}}$)
δ, δ^*	boundary layer and displacement thicknesses in inches or feet
θ	{ angle in degree or radians momentum thickness in inches or feet
Λ	see Eq. (3.2)
λ	constant or parameter (see Eq. 3.1)
τ_w	shear stress on wall in lbs/ft ²
ρ	density in slugs/ft ³
ω	circular oscillation frequency in rads/sec
ω_h, ω_α	translational and rotational natural frequencies respectively ($\omega_h^2 = K_h/m, \omega_\alpha^2 = K_\alpha / I_\alpha$)
μ	mass density ratio (see Eq. 2.2)
ν	kinematic viscosity in ft ² /sec

I. Introduction

A faired towline, a heavy-duty cable enclosed in a symmetrically streamlined body, is commonly used for towing naval underwater vessels, such as a Sonar device [1]*, and oceanographic instrumentations [2] by a surface vessel. This dynamical system consisting of the towing vessel, towline and towed body is essentially subject to the interaction of inertial, hydrodynamic and elastic forces. The dynamics of such a system is indeed a complex problem which in general involves the steady motion, the stability and the unsteady response under different environment for certain mode of operation. Progress has been made so far by one of the following approximations: isolating an individual component from the whole system, treating only certain preferred mode of motion, and/or decoupling the forces acting on the system. Whicker [3] derived the equations of motion for a cable-body system in planar motion in terms of "generalized", but empirical, loading functions. A mathematical solution of the unsteady forced motion based on the method of characteristics was obtained to facilitate the discussion of the propagation of disturbance along the cable. Asymptotic solutions for the perturbation frequencies were also carried out on the basis of the small disturbance theory. His solution for the equilibrium configuration in terms of the loading functions under a steady towing is available for practical applications and was recently extended by Clark [4]. Strandhagen [5] derived the equations of motion in a more general way by dividing a towline into a finite number of segments and no solution was attempted for the system of equations. In these treatments, only the inertial and hydrodynamic forces were considered and the degrees of freedom of dynamical motion were also limited in a certain way. The elastic force has generally been neglected due to the assumption either that a towcable is extremely flexible or that hydroelastic stability is a less serious problem for the range of speed considered at present as compared with the aeroelastic problem for modern aircrafts. The earlier work in England for aircraft-towed body and the experimental program conducted at David Taylor Model Basin have contributed significantly to the understanding of some phase of this dynamical problem.

*The number in [] refers to the listed references.

This preliminary study is focused on the stability problem of a towable for high-speed operation as outlined in [6,7]. The first phase of this study is a literature survey pertinent to the dynamical motion and the stability. The list of bibliography given in Appendix I consists of four parts: (A) theoretical and experimental work directly concerning a towable system or its components, (B) basic theories and experiments in the related hydroelastic problem, (C) the existing knowledge to predict or to acquire the flow field around a faired or unfaired towline, towing vessel, and towed body, and (D) the mechanics of wakes which is believed to be essential in the study of the stability and the dynamic loading as well as the unsteady response of the system. The second phase consists in an examination of the flow field around a faired towline isolated from the system under a steady towing and an investigation of its hydroelastic stability in the light of the existing theoretical and experimental evidence. As far as the flow field is concerned, the faired towline may be treated essentially as a symmetrical hydrofoil with a very large aspect ratio and a variable sweep along the cable. However, there are basic differences in their elastic properties. Although the theory of flow past a fully-wetted airfoil or hydrofoil is well developed, an analysis appropriate to a faired towable for the purpose of extending the meager experimental data of the drag coefficient (including the effect of the inclined angle) of a sectional fairing to a larger range of Reynolds number yet remains to be developed. The cavitating flow past a hydrofoil is still in the stage of active research. The study of the hydroelastic stability inevitably requires a detailed flow field around the cable. As will be discussed in details in the next section, the classical type of hydroelastic flutter is not likely to occur in the towable problem, except in the presence of cavitation; however, the self-excited vibration due to the separation of flow and viscous wakes at high-speed operation has to be investigated. Therefore, a procedure of numerical computation for the flow field around a faired section normal to the flow is suggested here. The potential flow is obtained by a direct method based on the thin airfoil theory which provides the pressure distribution on the surface of the body for a given shape of fairing. Then the viscous boundary layer over the surface is solved for various Reynolds number. The separation of flow from the body

may be determined and this condition is essential for the study of self-induced vibration. On the other hand, this scheme may be either employed to design a faired section or to extend the data of experimental measurement of drag coefficient outside the range of Reynolds numbers available for the purpose of determining the steady loading function. Due to the short span of time available for this study only the first part of numerical calculation was carried out for the AN/SQA-10 Fleet Type Towline with the USNUSL Sectional Fairing for its pressure distribution and the implication to the self-excited instability will be discussed.

Finally, this preliminary study is concluded with the suggestion for future research on the stability problem of a faired towline system.

II. Hydroelastic Stability of a Towcable

During the last few years, considerable work has been done to extend aeroelasticity to high-density fluid or liquid and hydroelasticity becomes a new branch of naval science. A comprehensive account of aeroelasticity may be found in the books by Bisplinghoff, et.al. [7] and by Fung [8]. The hydroelastic research, both theoretical and experimental, has so far been scattered around various technical journals. Henry, et.al. and Abramson, et.al.'s papers [9, 10] which appeared in the Journal of Ship Research in 1959 are believed to be the first published work on hydroelasticity. The hydroelastic stability is in general concerned with the phenomena of divergence and flutter. Divergence is a static instability due to the interaction of hydrodynamic and elastic forces and a divergence speed is defined to be the critical speed over which the hydrodynamic force exceeds the elastic restoring force for a specific mode of deformation. For a faired towline, it is generally designed in such a way that the fairing is more or less free to rotate about the "bare" cable transversely to the free stream and the sectional fairings provide a very weak restraint longitudinally along the cable. Therefore, a fully-wetted, faired towline is always statically stable for steady speed and for zero speed provided that certain conditions for the centers of gravity and buoyancy are satisfied. This implies that there is no divergence for a towline. Flutter is defined as the dynamic instability due to interaction of inertial, hydrodynamic and elastic forces and the critical speed at which this phenomenon occurs is termed as a flutter speed. For a system linear in its response to loading, its stability to infinitesimal disturbance should provide the complete characteristics of its flutter and the origin of the force producing this motion is irrelevant. On the other hand, for a non-linear system there exists a functional relation between the forcing force and its response. When the magnitude of the force increases the amplitude of motion it provides, the phenomenon is called self-excited.

Now let us consider a symmetrical two-dimensional body moving at a steady speed in a fluid at rest or a uniform stream past a symmetrical body which has a thickness-to-chord ratio between 1 and 0, i.e. the shape of the body varies between a circular cylinder and a flat plate of zero thickness. The flow past a circular cylinder may be characterized by Reynolds number as given in the following table [11]:

Reynold NumberFlow Description

0 - 3	Stoke's and Oseen's flow
5 - 10	A pair of fixed vortices of Flopp's type
40	The instability of vortices
90	The formation of Karman's vortex street
150 - 300	The formation of a shear layer from the point of separation
300	The irregular and turbulent wake

As for the range of Reynold number suitable for naval application a turbulent wake is inavoidable for a bluff body such as a circular cylinder. The instability induced by the unsteady wake is generally described by the dimensionless Strouhal number defined as $S = fd/U$ where the vortex shedding frequency f may be determined from S for a given Reynold number. The Strouhal number is also known as the reduced frequency of oscillation. Strouhal [12] first showed in 1878 that the Strouhal number $S = 1/6$ for a cylinder and later Kovasnay [13] and Roshko [14] determined for circular cylinder with viscous wakes:

$$S = 0.212 \left(1 - \frac{\lambda}{R}\right), \quad R = \frac{Ud}{\nu} \quad (2.1)$$

where $\lambda = 21.2$ for $40 < R < 150$ and $\lambda = 12.7$ for $150 < R < 5000$. For higher Reynolds number, the "body regime", the region between the point of separation and fully-developed turbulent wake, responsible for periodic oscillation becomes shorter. This "bare" cable represents a limiting case of a faired towline. The stability is indeed a serious problem and the use of bare cables should be avoided even for a moderate speed of operation. The study is outside the present scope. The other limiting case is a flat plate of zero thickness, which has a zero pressure gradient along the plate and a uniform velocity outside the boundary layer. An extensive account on this problem has been published in many texts, e.g. Schlichting's text [15]. A symmetrical body of finite thickness (including a thin plate) with non-zero pressure gradient along the surface and viscous wake behind the trailing edge is a practical model for present study. The hydroelastic instability or flutter of a thin symmetrical body in a steady, uniform stream may be classified as the classical type and the nonclassical type. In the analysis of the former the hydrodynamic force is derived essentially from the clean potentially flow, i.e. a

fully-wetted surface without a flow separation and viscous wake, while the latter involves the flow of a real fluid - the boundary layer and viscous wake.

(A) The Classical Flutter. Let us examine the classical flutter for a faired towline. Following Henry, et.al. [9] a non-dimensional parameter paramount in a flutter analysis is the so called "mass-density ratio", which is defined as

$$\mu = \frac{4m}{\pi \rho c^2} \quad (2.2)$$

where m is the mass of the faired towline per unit length, c the chord and ρ the density of liquid or water. Since water is roughly one thousand times denser than sea-level air, μ turns out to be of order of 1/2 for hydrofoils but 50 or more for aircraft wings. This difference works greatly in favor of submerged and fully-wetted naval vessels for dynamic stability. A summary of the flutter analysis is given as follows.

The system may be idealized by isolating a section from the towline as a rigid hydrofoil supported by two springs for the bending and torsional displacements, $h(t)$ and $\alpha(t)$, (see Fig. 1). The equations of motion for the system are given by

$$\begin{aligned} m\ddot{h} + S_{\alpha}\ddot{\alpha} + K_h h &= -L, & \dot{\alpha} &= \frac{d}{dt}, \\ S_{\alpha}\ddot{h} + I_{\alpha}\ddot{\alpha} + K_{\alpha}\alpha &= M. \end{aligned} \quad (2.3)$$

The flutter analysis is carried out by assuming the solutions for an undamped oscillation (or the neutral stability),

$$\begin{aligned} \alpha &= \bar{\alpha}_0 e^{i\omega t} \\ h &= \bar{h}_0 e^{i\omega t} \end{aligned} \quad (2.4)$$

where the flutter frequency ω is a real number, and $\bar{\alpha}_0$ and \bar{h}_0 are complex amplitude functions. The forcing functions, L and M_y , are the hydrodynamic force and moment for this periodic oscillation. For the classical flutter analysis, by assuming the small disturbances the forcing functions may be expressed as a linear function of the disturbances, namely α , h and their first and second derivatives, e.g. Theordosen [16]

and many others have shown that

$$L = \pi \gamma b^2 [\ddot{h} + U \dot{\alpha} - b a \ddot{\alpha}] + 2 \pi \rho U b C(k) [\dot{h} + U \alpha + b(\frac{1}{2} - a)\dot{\alpha}]$$

(2.5)

$$M_y = \pi \gamma b^2 [b a \ddot{h} - U b(\frac{1}{2} - a)\dot{\alpha} - b^2(1/8 + a^2)\ddot{\alpha}] + 2 \pi \rho U b^2 [\frac{1}{2} + a] C(k) [\dot{h} + U \alpha + b(\frac{1}{2} - a)\dot{\alpha}],$$

where $k = \omega b/U$, the reduced frequency, $C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + i H_0^{(2)}(k)}$, and

$H_1^{(2)}$ and $H_0^{(2)}$ are the Hankel functions of the second kind. Substituting (2.4) and (2.5) into (2.3), two linear homogeneous equations are obtained. In order to have the undamped periodic solutions assumed in (2.4), the determinant formed by the coefficients of \bar{h} and $\bar{\alpha}$ has to vanish as follows:

$$\left| \begin{array}{cc} \mu + 1 - \frac{i 2 C(k)}{k} - \mu \left(\frac{\omega h}{\omega_x}\right)^2 \left(\frac{\omega_x}{\omega}\right)^2, & \mu x - a - \frac{i}{k} [1 + (1-2a)C(k)] - \frac{2C(k)}{k^2} \\ \mu x - a + \frac{i}{k} [1 + 2a] C(k), & \mu x^2 [1 - \left(\frac{\omega h}{\omega_x}\right)^2] + a^2 + \frac{1}{8} - \frac{i}{k} [\frac{1}{2} - a + (2a^2 - \frac{1}{2}) C(k)] + (1 + 2a) \frac{C(k)}{k^2} \end{array} \right| = 0$$

(2.6)

This complex determinant is solved for the critical speed U_f and frequency ω at flutter with the hydrodynamic, elastic and geometric properties as parameters. On the basis of the dimensional argument discussed before, the mass density ratio μ plays a paramount role for flutter as indicated here in (2.6). Fig. 2 is a typical result of the theory and test data from Woolston and Castile [17]. As far as flutter is concerned the sweep angle and a large aspect ratio tend to lower the minimum value of μ ; however, the existing knowledge on hydroelastic flutter indicates that the asymptote is located at $\mu > 0.5$. The value of μ for the AN/SQA-10 Fleet Type Towline with the USNUSL Sectional Fairing is calculated as follows.

Chord of the fairing, $c = 0.75$ ft.

Salt water density, $\rho = 64/g$ slugs/ft³

Unit weight of armored electrical cable (SK43986A), $w_r = 2.58$ lbs/ft.

Unit weight of the fairing in air, $w_f = 5.16$ lbs/ft.

$$\mu = 4(w_r + w_f) / \rho g \pi c^2 = 4(5.16 + 2.58) / (64)(3.14)(0.75)^2 = 0.274.$$

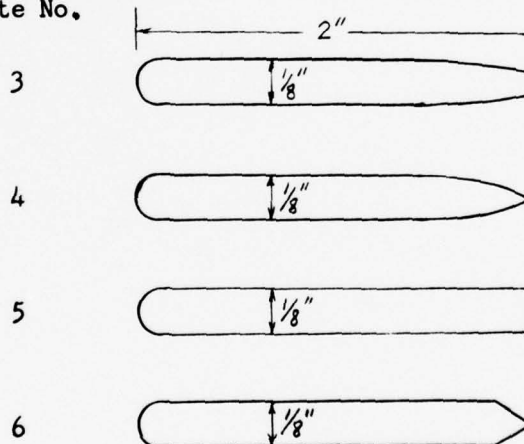
For this value of the density ratio, there exists no possibility of the classical flutter on the basis of the present state of the hydroelastic flutter analysis and of the experimental evidence.

(B) The Self-excited Vibration

For a viscous flow past a thin fairing, the existence of viscous wake responsible for the self-excited vibration [18, 19, 20] and the possibility of flutter due to cavitation [21] are deserved a further study of the flow field around a faired towline at a wide range of Reynolds number anticipated in modern naval operation. In the light of the experimental studies, the implication to the instability of a towline is explored here. The part of experimental results of [18, 19, 20] related to this problem is summarized and is discussed as follows.

(1) The result of Ippen, et.al. [18]:

The configuration: Plate No.



The thickness-to-chord ratio, $d/c = 1/16$

The approximate natural frequency: 45~100 cps in water

90~200 cps in air

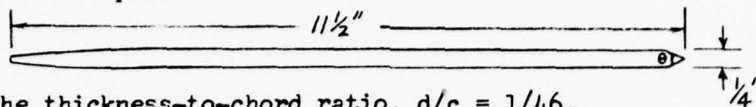
The maximum speed tested: 35 fps or 20.7 knots.

Plate No.	Observed Max. Amplitude (radians)	Strouhal No. at Max. Amplitude	Strouhal No. at Max. Frequency	Frequency at Max. Amplitude (cps)
4	0.003	-	-	165 380
3	0.0035	0.237	0.251	165 620 380
5	0.005	0.212	0.219	85, 175, 390 470, 530, 570, 670
6	0.016	0.146	0.174	95, 156, 205 285, 370, 680

A schematic plot of the characteristics of self-induced vibration is shown in Fig. 3. A linear relation for the Strouhal number with a distribution of discrete steps for the resonant response coincident with the natural frequencies of the elastic body was obtained from the experimental data. One of the important characteristics related to the instability problem, namely, the separation condition in the neighborhood of the tail was not carried out. However, the result indicated that there is no serious instability within the range of test speeds for Plate No. 4, which has a streamlined tail. The wake is primary due to the diffusion of the non-separated boundary layers detaching from both sides of the body at the trailing edge. The vorticity in this case is very weak; so even if the frequency coincides with the natural frequency of the plate, the vibration is insignificant. However, if higher speeds were available in the test, it is expected that the instability tends to have similar characteristics as Plate Nos. 3 and 6 as the boundary layers are separated. Then, as the speed increases, serious vibration will appear at the discrete natural frequencies of the plate in the spectrum of the exciting frequencies induced by the vorticity in the wakes. Another interesting result should be observed here that the vibration for Plate No. 5 is smaller than Plate Nos. 3 and 6 and possibly No. 4 when the flow is separated. This is due to the fact that for Plate No. 5, the points of separation are more or less fixed while the points of separation for other plates tend to oscillate and the oscillations of lift tends to amplify the vibration further. Should a fairing be designed for a supercavitating high-speed flow, the trailing edge geometry of Plate No. 5 is of particular interest.

(2) The result of Heskestad, et.al. [19]:

The test plate



The thickness-to-chord ratio, $d/c = 1/46$.

The range of test speed, 10 to 60 fps.

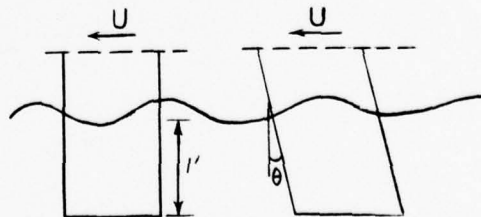
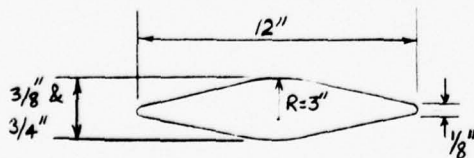
	Trailing Edge Geometry, θ				
	180°	90°	60°	45°	30°
Plate No.	1	6	7	8	9
Speed (fps)	16.5- 50.4	12.1- 59.7	16.2- 60.2	17.1- 55.3	54.8
Observed Frequencies (cps)	145- 580	100- 600	160- 470	103- 597	597
Strouhal No.	0.192	0.183	0.158	0.275	0.300
Distance Between Separation Point	d	d	d	d	none
Amplitude at * the Resonance Frequency of 260 cps	100	190	380	43	none

* Relative to 100 for Plate No. 1.

A typical result is reproduced here in Fig. 4, which is similar to those of Ippen, et.al. In addition, the observation of the distance between separation points confirmed that the separation of the boundary layers is essential prior to the inception of this instability.

(3) Gongwer [20]

The configuration



The angle of sweep, $0 \leq \theta \leq 30^\circ$.

The maximum speed, 45 knots for 3/8"-plate and 60 knots for 3/4" plate.

The step behavior at resonant appeared on the curve for the Strouhal number in the frequency/velocity plots is again confirmed by the experiment. For a trailing edge thickness less than 0.07 inch, no instability was observed in the speed range. The Strouhal number for the instability varies with the thickness of the trailing edge, however, if the width of the viscous wake in the fully developed region instead of the trailing edge thickness is used, the Strouhal number is approximately constant with a value of 0.18.

The result of these experiments may be summarized as the following:

- (1) The self-excited vibration may be characterized by the Strouhal number, which has a constant value for a given body configuration and elastic properties as indicated by the linear slope on the frequency versus velocity plot for each plate under a given restraint condition.
- (2) No definite limit at which this instability is incipient has been observed because all experiments are more or less emphasized on the response characteristics of the self-excited vibration.
- (3) No instability is observed for a thin plate with a streamlined tailing or a very small trailing edge angle. This evidence implies that within the range of speed tested, the flow is not separated from the body, then the forcing frequency by the vorticity in the viscous wake primarily due to the diffusion of the boundary layers detaching from the trailing edge is insignificant.
- (4) The resonant vibration occurs as the natural frequencies of the plate under a given restraint condition coincide with the frequency induced by the vorticity in the viscous wake. This behavior appears in the step-like curve in frequency/velocity plots.
- (5) The Strouhal number may be reduced approximately to a constant value for the self-excited vibration if the width of viscous wake instead of the thickness of body is used in the Strouhal number.

III. Flow Around Thin, Paired Towcables

Since the classical type of flutter is not likely to occur and only the incipient instability for the self-excited vibration is crucial for the operation of a tow cable, an analysis of the steady flow of real fluid around a fairing is desired at this stage. In addition, the result of this analysis may be used to extend the drag coefficient presently available from the test to a wider range of Reynolds number, which is essential in the study of the equilibrium characteristics of a steady towing. The steady flow has to be solved by an iterative scheme by incorporating the classical potential flow theory with the boundary layer calculation and also isolating the effect of sweep due to the inclination of the towline. Sears [22] has shown that for a straight cylinder in uniform flow the principle of independent flow is valid, i.e. the cross flow along the cable may be evaluated after the main flow is solved. Although the inclination angle might be much smaller than 90° and the curvature might not be small, it seems reasonable to separate this effect at this stage of analysis and treat it as a uniform flow past symmetrical hydrofoil with a very large aspect ratio.

The iterative analysis may be carried out as follows. First, solve the potential flow past a symmetrical hydrofoil, from which the pressure distribution along the surface is derived. Then, the boundary layer along the surface is calculated for a given Reynold number together with the displacement and momentum thickness and the wall shear stress. Finally, potential flow has to be solved again to account for the displacement thickness. The final result will provide the pressure distribution, which now dependent on Reynold number, may be used to predict the separation condition by recalculating the whole boundary layer. The drag coefficient may be obtained from the shear stress distribution on the surface. Of course, after this scheme has been set up the iteration may be carried forward for further refinement. As a check of the validity or accuracy of the analysis, the computed drag coefficient may be checked against the drag coefficient obtained from experiments or field measurement in the range of Reynold numbers available.

Potential Flow Solution

For a thin symmetrical hydrofoil, the thin airfoil theory or the linearized potential theory (see Thwaites [23]) is suitable except the troublesome singular behavior at the fore stagnation point and the trailing edge. In Appendix II, the calculation of this solution is carried out for the AN/SQA-10 Fleet Type towline with the USNUSL Sectional Fairing and the result is given in Fig. 5. However, a fairing with larger thickness-to-chord ratio, it is possible to use a nonlinear theory or simply by a source distribution on the surface instead of on the chord line as in the linearized theory.

Laminar Boundary Layer

Thwaites' method [24] for computing a laminar boundary layer is a simple and straightforward integration of the potential flow velocity on the surface,

$$\lambda = \frac{\theta^2 U'}{\nu} = 0.45 U' U^{-6} \int_0^{x'} U^5 dx \quad (3.1)$$

where x' is from the fore stagnation point along the surface of the fairing. And other characteristics such as δ , δ^* , H and τ_w may be computed from the functions provided by the method in the reference. A check of laminar separation may be predicted by [25] as

$$\lambda_{min} = \frac{\delta^3}{\nu} \frac{dU}{dx} \quad (3.2)$$

The relation for λ and λ is also given by [24].

The Transition

Since the critical Reynolds number for the instability of a laminar boundary layer has been established by the small perturbation theory [25], $Re_{critical} = \left(\frac{U\delta}{\nu}\right)_{critical}$ is found to depend on λ . Therefore, from the λ found in the laminar boundary layer, the point of instability, may be located and then the point of transition is determined from the given empirical relation based on the rate of amplification of turbulent eddies (see [15]).

Turbulent Boundary Layer

It is recommended that Truckenbrodt's method [25] for calculating

the turbulent boundary layer may be used. The momentum thickness θ and the shape factor H have to be calculated simultaneously starting from the transition point where θ is continuous while H has a finite drop. The separation may again be detected by checking the value of $H = 1.8$ to 2.4.

IV Conclusions and Recommendations

On the basis of the existing theory and experimental evidence in hydroelasticity related to the instability problem of a faired towline under steady motion examined in this preliminary study, the general conclusions are given in the following:

- (1) The classical flutter is not likely for a faired towline having a section similar to the AN/SQA-10 Fleet Type Towline with the USNUSL Sectional Fairing.
- (2) The instability due to self-excited vibration requires a further investigation since the existing theoretical and experimental results emphasize the response rather than the incipient problem. However, the result provides the necessary conditions for the self-excited vibration, namely, the lower limit of Strouhal number (or the Strouhal number based on the width of the wake) and the natural frequency of the fairing under the practical restraint condition.
- (3) The sufficient condition for the incipient of the self-excited instability, observed in those studies and examined here, is the separation of the boundary layers on the surface of the fairing to provide a strong viscous wake and the oscillation of the separation points themselves.
- (4) On the basis of an extremely simplified calculation of the characteristics of the turbulent boundary layer for the AN/SQA-10 Fleet Type towline with the USNUSL Sectional Fairing at a speed of 30 knots normal to the fairing, it is found that the turbulent boundary layer is not separated from the surface up to a point about $\frac{1}{2}$ inch from the trailing edge due to the singular behavior in the linearized potential flow solution. The pressure increases rapidly thereafter while the local velocity decreases; Equation (J) indicates that the value of Γ becomes rapidly smaller than -0.06 . However, even if the flow separates in the region the width of the viscous wake is expected to be thin (the same order of the distance between separation point) and therefore it is unlikely that this weak wake will cause the self-excited type vibration.
- (5) For a three-fold increase of the Reynolds number, as seen from the calculation in Appendix II for the prediction of separation, the conclusion in (4) is still valid.
- (6) The calculation of the unsteady hydrodynamic force acting on the fairing should be delayed until we find that a towline is possible to have

the self-excited vibration and should then be incorporated with the study of unsteady motion.

The following recommendations are made:

- (1) An iterative analysis based on the theory outlined should be carried out and then incorporated with a numerical program suitable for a high-speed digital computer in such a way that for any nonanalytic shape of a fairing the flow field in terms of the pressure, boundary layer characteristics may be calculated for any Reynolds number. Then, this program will serve as the means
 - (a) to predict the possibility of separation at a desired Reynolds number and
 - (b) to calculate the drag coefficient outside the range of Reynolds number available from the experiments.

This information is not only essential to the stability but also to the design or improvement of a fairing at high-speed operation and to solve the equilibrium configuration under steady towing.

- (2) An experimental program to determine the elastic properties of a number of the existing towcables, e.g. the natural frequency under various restraint conditions.
- (3) The research in unsteady flow past a fully-wetted and/or a super-cavitating symmetrical hydrofoil is of primary interest in many aspects of the towcable problem, e.g. the stability and response problem during unsteady operation.

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Appendix II
Prediction of Flow Separation from the Pressure
Distribution for the AN/SQA-10 Fleet Type Towline
with the USNUSL Sectional Fairing

The Pressure Distribution

For a thin symmetrical hydrofoil, the potential flow may be obtained for a given shape from the thin airfoil theory as outlined in [23]. The linearized solution consists of the symmetrical solution,

$$u_s - iv_s = \frac{U}{\pi} \int_0^c \frac{y_s'(\xi)}{z - \xi} d\xi, \quad y_s'(\xi) = \frac{dy_s}{d\xi}, \quad (A)$$

and the chamber solution,

$$u_c - iv_c = U \left(\frac{c}{z} - 1 \right)^{\frac{1}{2}} \left\{ \frac{1}{\pi} \int_0^c \frac{[y_c'(\xi) - \infty]}{z - \xi} \left(\frac{\xi}{c - \xi} \right)^{\frac{1}{2}} d\xi + \frac{cC}{c - \xi} \right\} \quad (B)$$

where the thickness function, $y_s(\xi)$, and the chamber function, $y_c(\xi)$, are given for $0 \leq \xi \leq c$, $z = x + iy$, c the chord and C is a real constant. For a symmetrical hydrofoil like the fairing with zero angle of incidence, only the symmetrical part of the solution is needed. It is equivalent to a source distribution on the chord line, $0 \leq x \leq c$ and $y = 0$ of the strength,

$$q(x) = 2Uy_s'(x) \quad (C)$$

proportional to the slope of the surface and with the condition of closure that

$$\int_0^c q(x) dx = 0. \quad (D)$$

Then, the pressure distribution may be derived as a coefficient,

$$c_p = \frac{P - P_\infty}{\frac{1}{2} \rho U^2} = - \frac{2u_s}{U} \quad (E)$$

and the velocity on the surface is $U_m = 1 - \frac{1}{2} c_p$.

Let $x = \frac{1}{2} c (1 + \cos \phi)$ and $\xi = \frac{1}{2} c (1 + \cos t)$, then on the surface of the fairing,

$$u_s = - \frac{1}{\pi} \int_0^\pi \frac{(2U/c) y_s'(t)}{\cos \phi - \cos t} dt \quad (F)$$

$$c_p = \frac{4}{\pi c} \int_0^\pi \frac{y_s'(t)}{\cos \phi - \cos t} dt \quad (G)$$

Finally, by Watson's Method [26], the numerical computation may be carried out from the following series:

$$u_m = - \left(\frac{2U}{c} \right) \csc \left(\frac{m\pi}{N} \right) \sum_{\substack{1 \leq p \leq N-1 \\ \text{odd } p}} N^{-1} \cot \left(\frac{p\pi}{2N} \right) (y'_{m+p} - y'_{m-p}) \quad (H)$$

or

$$(c_F)_m = \left(\frac{4}{c} \right) \csc \left(\frac{m\pi}{N} \right) \sum_{\substack{1 \leq p \leq N-1 \\ \text{odd } p}} N^{-1} \cot \left(\frac{p\pi}{2N} \right) (y'_{m+p} - y'_{m-p}) \quad (I)$$

in which N is the number of equal dimensions taken for $0 \leq x \leq c$, m the m^{th} point on the chord line and the slope of the surface $y'(\phi) = \frac{dy}{dx} \frac{dx}{d\phi} = -\frac{c}{2} \sin \phi y'(x)$ for $\phi = \frac{m\pi}{N}$. The convenience of this method lies on the fact that any numerically describable surface with continuous slope may be solved and the coefficients of the coefficients of the series for $N = 20$ are given in [23]. Even a fairing may be described exactly by a formula, however, after the displacement thickness due to the boundary layer is added to the surface for a revised computation of the pressure distribution it is not practical to approximate it to a new equation or an infinite series.

The pressure distribution for the AN/SQA-10 Fleet Type Towline with the USNUSL Sectional Fairing is calculated as given in Fig. 5. The pressure distribution shows the characteristics of a semicircular nose and then an approximately zero-gradient over the major portion of the flat mid-body. The adverse pressure gradient near the tail is the center of interest as far as the separation and the incipient instability of the self-excited vibration are concerned. Unfortunately the linearized solution does not provide a correct value due to its singular behavior at the fore stagnation point and the trailing edge and the only possible approximation can be made for the purpose of predicting a separation is by the dashed lines indicated in Fig. 5. Some sample data of computations are given in Tables 1 and 2.

The Separation

The formal, conventional method for prediction has to follow the method given in Section III. First, the solution of the laminar boundary equation is calculated in terms of the boundary layer, displacement and momentum thickness as well as the shape factor, then the point of transi-

tion is determined, and finally the turbulent boundary layer is solved in a way similar to the laminar boundary layer. In the process of calculation, the possibility of separation is checked both for the laminar and turbulent boundary layers. Due to the short span for this preliminary study, it is not possible to carry out the solution formally. A rough estimation is carried out here on the basis of the above procedure, but certain approximation has to be assumed in order to use some known results. The following analysis is based on Reference [25] as outlined in Section III. Lets consider a speed of 30 knots or 50.7 feet per second normal to the AN/SQA-10 Fleet Type Towline with the USNUSL Sectional Fairing. For a semicircular nose with the Reynolds number, $\frac{U_{\infty} d}{\nu} \approx 5.2 \times 10^5$, it is reasonably expected that the transition point is located in the neighborhood of a distance equal to one half of the thickness from the fore stagnation point and there is no possibility of a laminar separation (even for a circular cylinder at this Reynolds number there). This part of the laminar boundary layer occupies only about 8% of the chord, therefore, the analysis in the following is assumed to have a turbulent boundary layer starting from the fore stagnation point with a zero pressure gradient joint to the pressure distribution of the mid-body. We further assume that θ computed by the flat plate formula may be carried on to the region of adverse pressure gradient and the separation is determined by

$$\Gamma = \frac{\theta}{U} \frac{dU}{dx} \left(\frac{U}{\nu} \theta \right)^{\frac{1}{4}} < - 0.06 \quad (J)$$

where

$$\theta = 0.036 x \left(\frac{U_{\infty} x}{\nu} \right)^{-1/5} \text{ for } 5 \times 10^5 < R_c < 10^7 \quad (K)$$

and $R_c = U_{\infty} c / \nu = 3.15 \times 10^6$. Let $x = 8.5$ " chosen for the calculation and it is found that

$$\Gamma = - 0.045$$

Therefore, it is not expected the boundary layer to separate from the surface at a station $\frac{1}{2}$ inch from the trailing edge. In this approximation, θ is underestimated in the region of adverse pressure gradient and the choice of the station is quite arbitrary. If a separation occurs at a station, the thickness of the wake will have the same value approximately as the distance between the separation points. However, the calculation can-

not be carried too close to the trailing edge due to the singular behavior of the linearized potential flow solution.

Since Eq. (K) is good for $R_c = 10^7$, it is expected that the conclusion derived here is still valid if there is a three-fold increase in the chord length for a geometrically similar section.

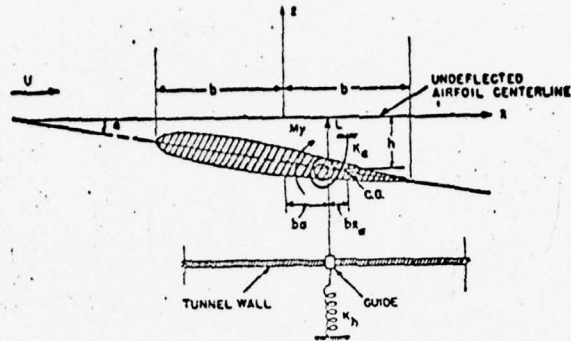


Fig. 1 Hydrofoil restrained by bending and torsional spring

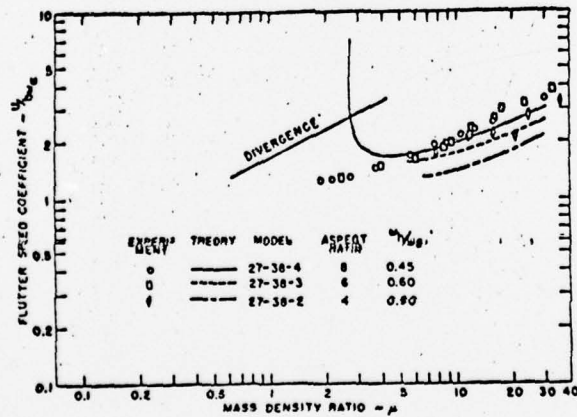


Fig. 2 Theoretical and measured plot of dimensionless flutter speed versus mass-density ratio for typical model in reference [17]

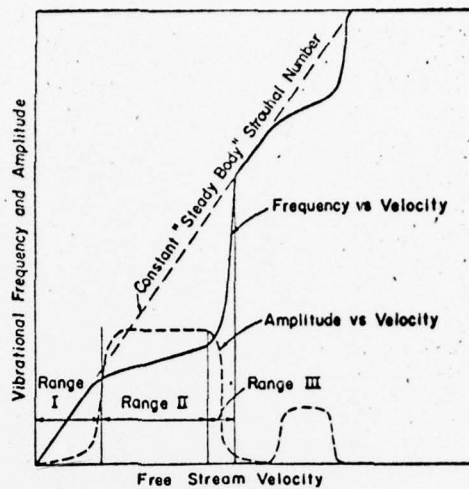


Fig. 3 Qualitative vibrational behavior

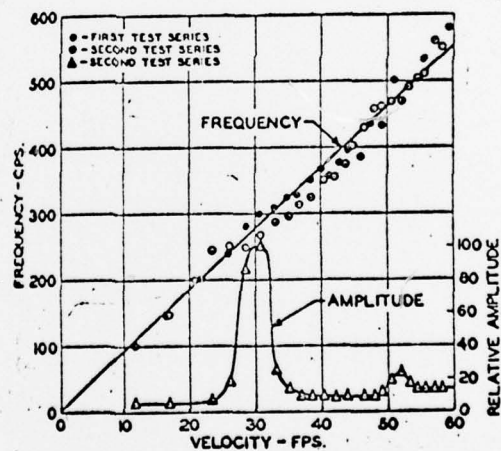
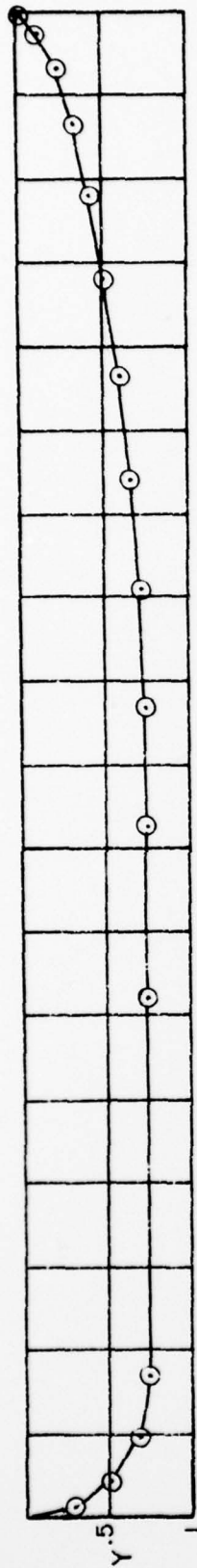


Fig. 4 Frequency and relative amplitude of vibration versus velocity



The Mid Section of the USNUSL Sectional Fairing (DTMB Hydrofoil Shape No. 7)

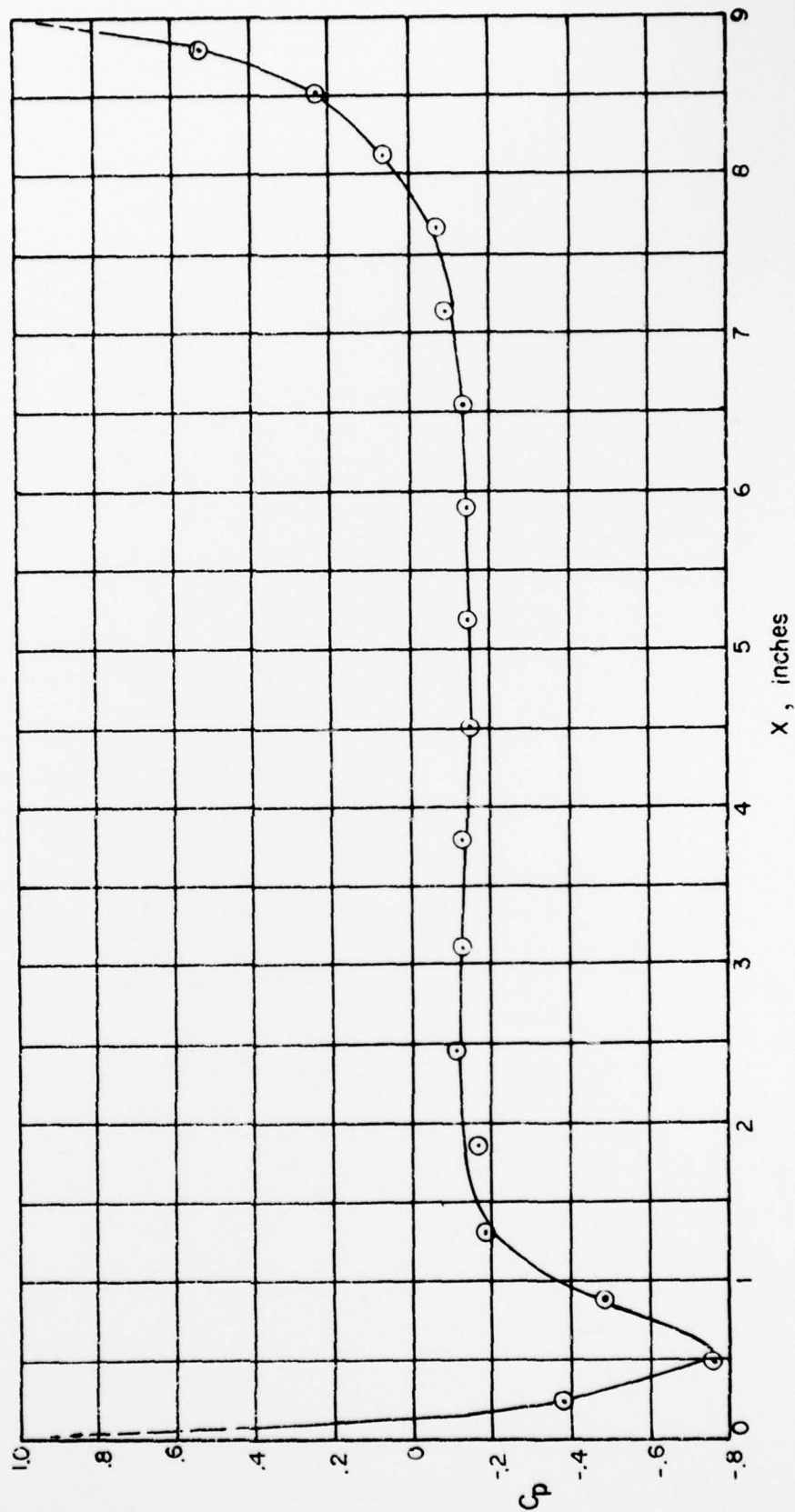


Fig. 5 The Pressure Distribution on the USNUSL Sectional Fairing

Table 1. The Slope of the Mid Section of the AN/SQA-10 Fleet Type
Towline with the USNUSL Sectional Fairing

No. of Point i	α (degrees)	cos	x/c	x^* (in.)	y_i	$y_{i+\frac{1}{2}}$	$y_{i-\frac{1}{2}}$	y_i
0	180	-1.0000	0.0000	0.0000	0.0000	—	—	—
	175.5	-.9969	.0016	.0144	.1463			
1	171	-.9877	.0066	.0594	.2926	.2677	1.704	
	166.5	-.9724	.0138	.1242	.4140			
2	162	-.9511	.0245	.2205	.5310	.2158	1.374	
	157.5	-.9239	.0381	.3429	.6298			
3	153	-.8910	.0545	.4905	.7043	.1152	0.0318	
	148.5	-.8526	.0737	.6633	.7450			
4	144	-.8090	.0955	.8595	.7500	.0050	0.0318	
	139.5	.7604	.1198	1.0782	.7500			
5	135	.7071	.1465	1.3185	.7500	.0000	0	
	130.5	.6495	.1753	1.5777	.7500			
6	126	.5878	.2061	1.8549	.7500	.0000	0	
	121.5	.5225	.2388	2.1492	.7500			
7	117	.4540	.2730	2.4570	.7500	.0000	0	
	112.5	.3827	.3087	2.7783	.7500			
8	108	.3090	.3455	3.1095	.7491	-.0004	-.0025	
	103.5	.2335	.3833	3.4497	.7496			
9	99	.1564	.4218	3.7962	.7482	-.0038	-.0242	
	94.5	-.0785	.4608	4.1472	.7458			
10	90	0	.5000	4.5000	.7390	-.0136	-.0866	
	85.5	+.0785	.5393	4.8537	.7322			
11	81	.1564	.5782	5.2038	.7189	-.0307	-.1827	
	76.5	.2335	.6168	5.55.2	.7035			
12	72	.3090	.6545	5.8905	.6801	-.0468	-.2980	
	67.5	.3827	.6914	6.2226	.6567			
13	63	.4540	.7270	6.5430	.6245	-.0645	-.4106	
	58.5	.5225	.7613	6.8517	.5922			
14	54	.5878	.7939	7.1451	.5522	-.0800	-.5093	
	49.5	.6495	.8248	7.4232	.5122			
15	45	.7071	.8536	7.6824	.4689	-.0867	-.5520	
	40.5	.7604	.8802	7.9218	.4255			
16	36	.8090	.9045	8.1405	.3794	-.0922	-.5869	
	31.5	.8526	.9263	8.3367	.3333			
17	27	.8910	.9455	8.5095	.2858	-.0950	-.6047	
	22.5	.9239	.9619	8.6571	.2383			
18	18	.9511	.9756	8.7804	.1851	-.1065	-.6779	
	13.5	.9724	.9862	8.8758	.1318			
19	9	.9877	.9939	8.9451	.0736	-.1165	-.7416	
	4.5	.9969	.9984	8.9856	.0153			
20	0	1.0000	1.0000	9.0000	0.0000	—	—	—

* x = the distance from the fore stagnation point in inches

Table 2. A Sample Calculation of C_p at $x = 0.0594$ inch from the Fore Stagnation Point (See Equation 1)

y'_{1+p}	y'_{1-p}	$y'_{1+p} - y'_{1-p}$	$C_{p1}/\frac{1}{c} \csc(\pi/20)$	C_{p1}
1.374		1.374	.8729	
.0318	-1.374	1.4058	.2928	
.0000	-.0318	.0318	.0038	
-.0083	.0000	-.0083	-.0007	
-.0866	.0083	-.0949	-.0056	
-.2979	.0866	-.3845	-.0164	
-.5169	.2979	-.8148	-.0249	
-.5869	.5169	-1.1038	-.0228	
-.6779	.5869	-1.2648	-.0152	
—	.6779	-.6779	-.0026	
			1.0813	3.072